Interval-based Robustness of Linear Parametrized Filters

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Introduction. This article deals with the resilient implementation of parametrized linear filters (or controllers), i.e. with realizations that are robust with respect to their fixed-point implementation.
The implementation of a linear filter/controller in an embedded device is a difficult task due to numerical deteriorations in performances and characteristics. These degradations come from the quantization of the embedded coefficients and the roundoff occurring during the computations.

As mentioned in [1], there are an infinity of equivalent possible algorithms to implement a given transfer function $h$. To cite a few of them, one can use direct forms, state-space realizations, $\rho$-realizations, etc. Although they do not require the same amount of computation, all these realizations are equivalent in infinite precision, but they are no more in finite precision. The optimal realization problem is then to find, for a given filter, the most resilient realization.

We here consider an extended problem with filters whose coefficients depend on a set $\theta$ of parameters that are not exactly known during the design. They are used for example in automotive control, where a very late fine tuning is required.

Linear parametrized filters. Following [3], we denote $Z(\theta)$ the matrix containing all the coefficients used by the realization, $h_{Z(\theta)}$ the associated transfer function and $\theta^q$ the quantized version of $\theta$. $Z^q(\theta^q)$ is then the set of the quantized coefficients, i.e. the quantization of coefficients $Z(\theta^q)$ computed from the quantized parameters $\theta^q$. The corresponding transfer function is denoted $h_{Z^q(\theta^q)}$. 

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Performance Degradation Analysis. The two main objectives of this article are to evaluate the impact of the quantization of $\theta$ and $Z(\theta)$ on the filter performance and to estimate the parameters $\theta$ that give the worst transfer function error in the set of possible parameters $\Theta$.

For that purpose, there are mainly two kinds of tools to study the degradation of filter performance due to the quantization effect: 

1) use a sensitivity measure (with respect to the coefficients) based on a first order approximation and a statistical quantification error model;

2) use interval tool, based on transfer function with interval coefficients. In both cases, we seek the maximal distance between the exact transfer function $h_Z(\theta)$ and the quantized one $h_Z(\theta^\dagger)$. For that purpose, we can use the $L_2$-norm i.e., $\|g\|_2^2 \triangleq \sqrt{\frac{1}{2\pi} \int_0^{2\pi} |g(e^{j\omega})|^2 d\omega}$ or the Maximum norm i.e., $\|g\|_\infty \triangleq \max_{\omega \in [0, 2\pi]} |g(e^{j\omega})|$.

The measure of the degradation of the finite precision implementation is then given by $\|h_Z(\theta) - h_Z^\dagger(\theta^\dagger)\|_\diamond$, with $\diamond \in \{2, \infty\}$. So the worst-case parameters $\theta_0$ can be found by solving:

$$\arg \max_{\theta \in \Theta} \|h_Z(\theta) - h_Z^\dagger(\theta^\dagger)\|_\diamond.$$ (1)

Since $\Theta$ is an interval vector, we denote $[h]$ the interval transfer function. With an interval approach, we can define the following constrained global optimization problem:

Maximize $\|[h]^\dagger_Z(\theta^\dagger) - [h]_Z(\theta)\|_\diamond$ subject to $\theta \in \Theta$. (2)

Note that in both cases, the evaluation of the norms can be done in interval with $\omega \in [0, 2\pi]$.

We will present the solutions of this problem using interval optimization methods [2] and we will compare them with the statistical sensitivity approach.

References:

