Low Parametric Sensitivity realization design for FWL implementation of MIMO Controllers

Theory and application to the active control of vehicle longitudinal oscillations

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CAO'06 - 26-28 April 2006 - Cachan France

Context

- Linear Time Invariant filters or controllers
- Finite Word Length implementation of control algorithms

Motivation

- Evaluate the impact of the quantization of the embedded coefficients
- Compare various realizations and find an *optimal* one

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FWL degradation

Origin of the degradation

The deterioration induced by the FWL implementation comes from :

- Quantization of the involved coefficients
  $\longrightarrow$ *parametric errors*
- Roundoff noises in numerical computations
  $\longrightarrow$ *numerical noises*

Only the deterioration induced by the quantization of coefficients is considered here.

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Equivalent realizations

Let's consider a transfer function $H(z)$ and one of its realization (A_q, B_q, C_q, D_q)

$$H(z) = C_q(zI - A_q)^{-1}B_q + D_q$$

$$\left\{ \begin{array}{rcl} qX_k & = & A_qX_k + B_qU_k \\ Y_k & = & C_qX_k + D_qU_k \end{array} \right. \quad \text{with } qX_k \triangleq X_{k+1}$$

The realizations of the form $(T^{-1}A_qT, T^{-1}B_q, C_qT, D_q)$, with T a non-singular matrix, are all equivalent in infinite precision.
They are no more in finite precision.

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Transfer function sensitivity measure

Gevers and Li (1993) have proposed a measure of the sensitivity of the transfer function with respect to the coefficients A, B and C

$$M_{L_2} \triangleq \left\| \frac{\partial H}{\partial A} \right\|_2^2 + \left\| \frac{\partial H}{\partial B} \right\|_2^2 + \left\| \frac{\partial H}{\partial C} \right\|_2^2$$

The optimal design problem consists in finding

$$\underset{T \text{non singular}}{argmin} \quad M_{L_2}(T^{-1}AT, T^{-1}B, CT, D)$$

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Outline

The need of a unifying framework

Various implementation forms have to be taken into consideration

- shift-realizations
- δ-realizations
- observer-state-feedback
- direct form I or II
- cascade or parallel realizations
- etc...

The need of a unifying framework

In order to encompass all these implementations, we have proposed a specialized implicit state-space realization to be used as a unifying framework :

Interests

- macroscopic description of a FWL implementation
- more general than previous realizations
- more realistic with regard to the parameterization
- directly linked to the in-line computations to be performed

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Implicit State-Space Framework

The control algorithm is described with

1. $J.T_{k+1} = M.X_k + N.U_k$
2. $X_{k+1} = K.T_{k+1} + P.X_k + Q.U_k$
3. $Y_k = L.T_{k+1} + R.X_k + S.U_k$

Intermediate variables computation

Implicit State-Space Framework

$$\begin{pmatrix} J & 0 & 0 \\ -K & I & 0 \\ -L & 0 & I \end{pmatrix} \begin{pmatrix} T_{k+1} \\ X_{k+1} \\ Y_k \end{pmatrix} = \begin{pmatrix} 0 & M & N \\ 0 & P & Q \\ 0 & R & S \end{pmatrix} \begin{pmatrix} T_k \\ X_k \\ U_k \end{pmatrix}$$

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State-vector computation

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Output computation

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Intermediate variables

The intermediate variables introduced allow to

- make explicit all the computations done
- show the order of the computations
- express a larger parameterization

Examples

A realization with the δ-operator is described by :

$$\begin{cases} \delta X_k = A_\delta X_k + B_\delta U_k \\ Y_k = C_\delta X_k + D_\delta U_k \end{cases} \qquad \delta \triangleq \frac{q-1}{\Delta}$$

and it corresponds to the following implicit state-space :

$$\begin{pmatrix} I & 0 & 0 \\ -\Delta I & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} T_{k+1} \\ X_{k+1} \\ Y_k \end{pmatrix} = \begin{pmatrix} 0 & A_\delta & B_\delta \\ 0 & I & 0 \\ 0 & C_\delta & D_\delta \end{pmatrix} \begin{pmatrix} T_k \\ X_k \\ U_k \end{pmatrix}$$

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Examples

The Observer State-Feedback

$$\begin{cases} \hat{X}_{k+1} &= A_p\hat{X}_k + B_pU_k + K_f(Y_k - C_p\hat{X}_k) \\ U_k &= -K_c\hat{X}_k + Q(Y_k - C_p\hat{X}_k) \end{cases}$$

where (A_p, B_p, C_p) corresponds to the plant system and K_c, K_f and Q are the controller's parameters.

A first parametrization

$$\begin{pmatrix} \begin{pmatrix} I & 0 \\ -Q & I \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -K_f & -B_p \end{pmatrix} & I & 0 \\ \begin{pmatrix} 0 & -I \end{pmatrix} & 0 & I \end{pmatrix} \begin{pmatrix} \begin{pmatrix} T^{(1)}_{k+1} \\ T^{(2)}_{k+1} \end{pmatrix} \\ \hat{X}_{k+1} \\ U_k \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} -C_p \\ -K_c \end{pmatrix} & \begin{pmatrix} I \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \end{pmatrix} & A_p & 0 \\ \begin{pmatrix} 0 & 0 \end{pmatrix} & 0 & 0 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} T^{(1)}_{k} \\ T^{(2)}_{k} \end{pmatrix} \\ \hat{X}_{k} \\ Y_k \end{pmatrix}$$

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An other possible parametrization

$$\begin{pmatrix} I & 0 & 0 \\ -B_p & I & 0 \\ -I & 0 & I \end{pmatrix}\begin{pmatrix} T_{k+1} \\ \hat{X}_{k+1} \\ U_k \end{pmatrix} = \begin{pmatrix} 0 & -(QC_p + K_c) & Q \\ 0 & (A_p - K_f C) & K_f \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} T_k \\ \hat{X}_k \\ Y_k \end{pmatrix}$$

Outline

1. The classical low sensitivity realization problem
2. Macroscopic representation of algorithms through the implicit state-space framework
3. **The transfer function sensitivity measure**
4. The optimal realization design problem
5. Conclusion and Perspectives

Transfer function sensitivity measure

The sensitivity of the realization considered according to each coefficient involved

First sensitivity measure

$$M_{L_2}^1 \triangleq \sum_{X \in \{J,K,L,M,N,P,Q,R,S\}} \left\| \frac{\partial \tilde{H}}{\partial X} \right\|_2^2$$

with $\tilde{H}(z) \triangleq H(z) - D = C(zI - A)^{-1}B$.

$\tilde{H}$ is strictly proper

$\frac{\partial D}{\partial X}$ is independent of the state-space coordinate

Transfer function sensitivity measure

- Trivial parameters have not to be considered
  - 0, ± 1 : in the implicit form, numerous coefficients are null or equal to 1
  - Some coefficients (power of 2, ...) can be exactly implemented
- So, to a realization matrix X (J, K,..., S), a weighting matrix W_X is required

$$(W_X)_{i,j} = \begin{cases} 0 & \text{if } X_{i,j} \text{ could be \textit{exactly} implemented} \\ 1 & \text{else} \end{cases}$$

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Transfer function sensitivity measure

Weighted sensitivity measure in SISO

For a SISO transfer function H, with realization $\mathcal{R} = (J, K, L, M, N, P, Q, R, S)$, the sensitivity measure is

$$M_{L_2}^W \triangleq \sum_{X \in \{J,K,L,M,N,P,Q,R,S\}} \left\| \frac{\partial \tilde{H}}{\partial X} \times W_X \right\|_2^2$$

It can also be express as

$$M_{L_2}^W = \left\| \frac{\partial \tilde{H}}{\partial Z} \times W_Z \right\|_2^2 \quad \text{with} \quad Z \triangleq \begin{pmatrix} -J & M & N \\ K & P & Q \\ L & R & S \end{pmatrix}$$

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Transfer function sensitivity measure

In the MIMO case, $\frac{\partial \tilde{H}}{\partial X}$ and W_X are not the same size anymore. It is possible to introduce overall *sensitivity matrices* defined by

$$\left(\frac{\delta \tilde{H}}{\delta X}\right)_{i,j} \triangleq \left\| \frac{\partial \tilde{H}}{\partial X_{i,j}} \right\|_2$$

Weighted sensitivity measure in MIMO

the sensitivity measure is defined by

$$M_{L_2}^W = \left\| \frac{\delta \tilde{H}}{\delta Z} \times W_Z \right\|_F^2$$

where $\|.\|_F$ is the Frobenius norm.

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$\frac{\partial \tilde{H}}{\partial Z}$ or $\frac{\delta \tilde{H}}{\delta Z}$ can be expressed thanks to the following transfer functions

$$\begin{aligned} H_1(z) &= C(zI_n - A)^{-1} \\ H_2(z) &= (zI_n - A)^{-1}B \\ H_3(z) &= H_1(z)KJ^{-1} + LJ^{-1} \\ H_4(z) &= J^{-1}MH_2(z) + J^{-1}N \end{aligned}$$

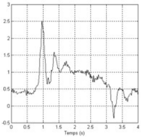
More details about this technical point in the paper.

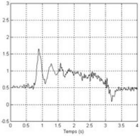
Outline

Active Control of Vehicle Longitudinal Oscillations

The example used here is an active control of longitudinal oscillations studied by (D. Lefebvre - PSA / P. Chevrel - EMN).

The first torsional mode (resonance in the elastic parts) which produces unpleasant (0 to 10 Hz) longitudinal oscillations of the car (*shuffle*), can be reduced by means of a controller acting on the engine torque.





The model of the powertrain was modeled in continuous-time form, and a continuous-time H_∞ optimal controller was designed (D. Lefebvre - PSA / P. Chevrel - EMN).

The discretized controller is defined by the transfert function

$$H(z) = \frac{-0.214z^{10} + 1.332z^9 - 3.402z^8 + 4.265z^7 - 1.803z^6 - 2.23z^5 + 4.105z^4 - 3.072z^3 + 1.285z^2 - 0.2948z + 0.02914}{z^{10} - 6.205z^9 + 16.34z^8 - 23.14z^7 + 17.51z^6 - 3.82z^5 - 5.545z^4 + 6.323z^3 - 3.294z^2 + 0.9679z - 0.1328}$$

Classical State-Space

We can first study classical state-space realizations.

$$Z_0 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & A_0 & B_0 \\ \cdot & C_0 & D_0 \end{pmatrix}$$

And we can consider each realization

$$Z(T) = \begin{pmatrix} I_q & & \\ & T^{-1} & \\ & & I_p \end{pmatrix} Z_0 \begin{pmatrix} I_q & & \\ & T & \\ & & I_m \end{pmatrix}$$

with T non singular

Classical State-Space

The *optimal design problem*, for the classical state-space, consists in finding

$$T_{opt} = \underset{T \text{ non singular}}{arg\ min} \; M_{L_2}^{W}\left(Z(T)\right)$$

This can be achieved thanks to a global optimization algorithm : the *Adpative Simulated Annealing* (ASA).

Results

realization	$M_{L_2}^{W}$	Nb parameters
companion form	$1.78e{+}14$	20
balanced form	81.44	120
optimal form	5.99	120

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State-Feedback-Observer structure

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It exists many equivalent state-feedback-observer realizations, using different state-feedback and observer gains.
They are all linked by Riccati equations.

In this example, 120 realizations are admissible. They correspond to different partitions of the closed-loop poles between state-feedback and observer dynamics.

State-Feedback-Observer structure

For the first observer-state-feedback form

$$Z = \begin{pmatrix} \begin{pmatrix} I & 0 \\ -Q & I \end{pmatrix} & \begin{pmatrix} -C_p \\ -K_c \end{pmatrix} & \begin{pmatrix} I \\ 0 \end{pmatrix} \\ \begin{pmatrix} -K_f & -B_p \end{pmatrix} & A_p & 0 \\ \begin{pmatrix} 0 & -I \end{pmatrix} & 0 & 0 \end{pmatrix}$$

we can evaluate the sensitivity :

- large diversity of numerical conditionning
- $M_{L_2}^W$ vary from $1.358e{+}2$ to $3.797e{+}8$
- we can choose the optimal partition (different from the usual partition)

State-Feedback-Observer structure

For the second observer-state-feedback form

- $M_{L_2}^{W}$ vary from $1.423e{+}2$ to $3.798e{+}8$
- results are similar to the first form (the best partitions for the first form are the best for the second)

CAO'06

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Outline

Conclusions and Perspectives

Conclusions

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- A transfer function sensitivity measure
- optimal design on various forms

Perspectives

- Other structurations to study (q/δ mixed realizations, ...)
- Multi-criteria optimization (Roundoff noise gain, stability related measure, ...)
- Toolbox to solve theses problems

Conclusions and Perspectives

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Acknowledgement

The authors wish to thank PSA Peugeot Citroën for their interest and financial support and Damien Lefebvre (PSA) for its numerical example.

Bibliography

M. Gevers and G. Li.
Parametrizations in Control, Estimation and Filtering Probems.
Springer-Verlag, 1993.

D. Alazard, C. Curres, P. Apkarian, M. Gauvrit, and G. Ferreses.
Robustesse et Commande Optimale.
Cepadues Edition, 1999.

T. Hilaire, P. Chevrel, and Y. Trinquet
Implicit state-space representation : a unifying framework for FWL implementation of LTI systems
IFAC05 Wolrd Congress, July 2005.

. Lefebvre, P. Chevrel and S. Richard.
An H_∞ based control design methodology dedicated to active control of longitudinal oscillations.
Proceedings of the Conference on Decision and Control, 2001.